PHYSICAL REVIEW B **82**, 037104 (2010)

Reply to "Comment on 'Poynting vector, heating rate, and stored energy in structured materials: A first-principles derivation' "

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(Received 19 May 2010; published 26 July 2010)

I argue that the form of the macroscopic Poynting vector derived in my paper [M. G. Silveirinha, Phys. Rev. B **80**, 235120 (2009)] is self-consistent with the usual form of the microscopic Poynting vector in nonmagnetic media, and that therefore there is no arbitrariness in the definition of the macroscopic Poynting vector as long as the usual expression for the microscopic Poynting vector is accepted as valid. I also emphasize that the macroscopic Poynting vector derived in my paper is coincident with the most general form of the Poynting vector reported in the literature for reciprocal stationary spatially dispersive media.

DOI: 10.1103/PhysRevB.82.037104 PACS number(s): 42.70.Qs, 78.20.Ci, 41.20.Jb

In the Comment,[1] Richter *et al.* argued that $\mathbf{s}=\mathbf{e}\times\mathbf{b}/\mu_0$ can be regarded as a completely general energy flux vector in bounded media, and acknowledged and clarified that the results of their letter[2] are valid if and only if the electromagnetic fields are not averaged in any sense. I certainly agree with Richter *et al.* that the energy conservation theorem derived in their work[2] is correct as long as one considers only "microscopic" electromagnetic fields, and that if that was the intended scope of their work, my considerations in Ref. 3 were inadequate.

However, I would like to point out that from the point of view of the study of realistic macroscopic systems the result of Richter *et al.* has a limited application, because in practice it is very difficult or even impossible to determine the microscopic fields in a system containing an extremely large number of atoms, molecules, or—in the case of metamaterials—inclusions. Thus, the true challenge is to determine an energy conservation theorem valid at the *macroscopic level*, i.e., when the system is described using an effective-medium model (the usual framework used to model the propagation of electromagnetic waves in matter or metamaterials). Such a problem is far from trivial, and this is why there are indeed valid reasons for the "many doubts expressed on under which conditions (static magnetic fields, stationary situations, dispersive media) Poynting theorem applies and on how Poynting's energy flux vector has to be interpreted," even though the authors of Ref. 2 did not see any.

In Ref. 3 I tried to shed some light on the definition of the Poynting vector in macroscopic systems. My goal was to demonstrate that under some conditions it is possible to define the macroscopic Poynting vector *self-consistently* with the conventionally adopted form for the microscopic Poynting vector in nonmagnetic media, which for the case of time harmonic excitation reads (here I use the same notations as in Ref. 3)

$$\mathbf{s}=\frac{1}{2}\text{Re}\left\{\mathbf{e}\times\frac{\mathbf{b}^*}{\mu_0}\right\}. \tag{1}$$

Specifically, I considered a periodic structured metamaterial formed by regular dielectric and metallic particles (with no intrinsic magnetism), and I showed that if the electromagnetic energy flux is indeed described at the microscopic level by Eq. (1) (which is exactly the form advocated by the authors of Ref. 1) then, in the absence of loss, the spatially averaged Poynting vector can be expressed as a function of the macroscopic fields and of the nonlocal dielectric function by the following exact relation:

$$\mathbf{S}_{\text{av},l}=\frac{1}{2}\text{Re}\left\{\left(\mathbf{E}_{\text{av}}\times\frac{\mathbf{B}_{\text{av}}^*}{\mu_0}\right)_l\right\} - \frac{\omega}{4}\mathbf{E}_{\text{av}}^*\cdot\frac{\partial\bar{\bar{\varepsilon}}_{eff}}{\partial k_l}(\omega,\mathbf{k})\cdot\mathbf{E}_{\text{av}} \quad (l=x,y,z), \tag{2}$$

consistent with a well-known textbook formula for the Poynting vector in macroscopic spatially dispersive media.[4,5] In the particular case where the effective medium has a local response, so that the effective dielectric function $\bar{\bar{\varepsilon}}_{eff}(\omega,\mathbf{k})$ is a quadratic function of the wave vector $\mathbf{k}$ such that (for simplicity here I restrict the discussion to nongyrotropic media; $\bar{\bar{\varepsilon}}_r$ and $\bar{\bar{\mu}}_r$ represent the local relative permittivity and permeability, respectively)

$$\frac{\bar{\bar{\varepsilon}}_{eff}}{\varepsilon_0}(\omega,\mathbf{k})=\bar{\bar{\varepsilon}}_r+\frac{c^2}{\omega^2}\mathbf{k}\times(\bar{\bar{\mu}}_r^{-1}-\bar{\bar{\mathbf{I}}})\times\mathbf{k}, \tag{3}$$

then the macroscopic Poynting vector can be written as

$$\mathbf{S}_{\text{av}}=\frac{1}{2}\text{Re}\{\mathbf{E}_{\text{av}}\times\mathbf{H}^*\}, \tag{4}$$

where $\mathbf{H}_{\text{av}}\equiv\mu_0^{-1}\bar{\bar{\mu}}_r^{-1}\cdot\mathbf{B}_{\text{av}}$ is the macroscopic magnetic field. Equation (4) is coincident with the well-known textbook formula for the macroscopic Poynting vector in magnetodielectric media, and enables one to characterize the flux of electromagnetic energy in a metamaterial in a very simple and convenient manner, without requiring any detailed knowledge of the microscopic fields, thereby greatly simplifying the modeling of electromagnetic propagation. In this sense, I find unjustified the claim of the authors of Ref. 1 that my theory is rather restricted rather than a general one, since as mentioned before the framework of macroscopic electromagnetism is often the only tool on which one can rely to model a complex system formed by many particles, and because Eq. (1) is coincident with the most general form for the Poynting vector in reciprocal stationary spatially dispersive media reported in the literature.[4,5]

I also want to make clear that I never claimed that Eq. (4)

corresponds to a "generally valid energy flux vector." Quite the contrary, as mentioned above, Eq. (4) is only valid in the absence of loss and provided the material's response can be described by polarization and magnetization vectors linked to the macroscopic fields through local relations. In the general case of a lossy material, it is not possible to relate the spatially averaged microscopic Poynting vector with the macroscopic fields (at least through an exact mathematical relation).

Finally, I would also like to comment on the assertion of the authors of Ref. 1 that "there is no 'correct' form [for the Poynting vector], as is pointed out nicely in Ref. 8. The choice is open and may depend on the system considered, the quantities of interest, and the calculations to be made." I personally have a different perspective on this matter because in my understanding the Poynting vector has a profound physical meaning and may be regarded as a local flux of electromagnetic energy at *a point*. Of course, one can discuss and debate if the usual form for the microscopic Poynting vector (in a vacuum) is consistent with such a property or not.[6] For example, Feynman *et al.* in Ref. 7 wrote that most likely the energy flow at a point is given by the usual form of the Poynting vector, even though there is no conclusive proof of that property. What was shown in Ref. 3 is that if $\mathbf{s}=1/2\,\mathrm{Re}\{\mathbf{e}\times\mathbf{b}^*/\mu_0\}$ may be regarded as the electromagnetic flux of energy at a point (in nonmagnetic media), then the spatially averaged electromagnetic flux of energy can be written in terms of the macroscopic fields as in Eq. (2). Therefore, there is no arbitrariness in the definition of macroscopic Poynting vector in Ref. 3, and most importantly it is self-consistent with the definition of the microscopic Poynting vector. I would also like to note that alternative definitions for the Poynting vector in macroscopic media [instead of Eq. (4)] typically imply different boundary conditions for the macroscopic fields at the interfaces, which in general are not compatible with the classical Maxwellian boundary conditions.

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